

## CHAPTER XIX

### LOGIC AND NATURAL SCIENCE:

#### FORM AND MATTER

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IT is commonplace that logic is concerned in some sense with form rather than with matter. Such words as “and, or, any, only, none, all, if, then, is and is not” are not material constituents of propositions. They express ways in which material is arranged for logical purposes, no matter how “logical” is defined. Such sentences as “John loved Mary” and “Peter disliked Joan” have the same form but different material contents, while “Two plus two equals four,” and “The sum of three interior angles of a triangle equal two right angles” are of the same form in spite of the difference of material content.<sup>1</sup> Again, the proposition “Carnegie is wealthy” and “Millionaires are wealthy” are of different forms, since the first proposition is about a singular as one of a kind, and the other is about a relation of kinds.

The intrinsic place of form in logical subject-matter is more than a commonplace. It states the character which marks off logical subject-matter from that of other sciences. It provides the fundamental postulate of logical theory. Recognition of this fact does not, however, settle the question of what the relation of form and matter is; whether there is any relation, what it is, or whether there is complete absence of relation. This problem is so fundamental that the way in which it is dealt with constitutes the basic ground of difference among logical theories. Those which hold there is no relation between form and matter are formalistic. They differ among themselves; some hold the doctrine that forms constitute a realm of metaphysical possibilities; others that forms are syntactical

<sup>1</sup>“Material” as used in the connection above is not to be identified with *existentially* material. Conceptual subject-matter is material in a non-existential proposition.

relations of words in sentences. The opposed type of logical theory holds that forms are forms-of-matter. The differential trait of the variety of this type of theory expounded in this book is that logical forms accrue to subject-matter in virtue of subjection of the latter in inquiry to the conditions determined by its end—institution of a warranted conclusion.

1. *Introduction.* There is no need to repeat or summarize here the arguments that have been adduced in support of this position. It is pertinent, however, to repeat, with some expansion, a point earlier made; namely, that the idea in question (that forms accrue to material which did not possess them in its original form) is a *vera hypothesis*, not a conception invented to serve the *ad hoc* need of a special logical theory. There are many instances in which original crude material takes on definitive form because of operations which order that material so that it can subserve a definite end. Indeed, this sort of thing happens wherever original raw materials are rearranged to meet requirements imposed by use of them as means to consequences. The supervening of form upon matter did not await the rise of logic. It would be truer, on the contrary, to say that logic itself had to wait until various arts had instituted operations by means of which crude materials took on new forms to adapt them to the function of serving as means to consequences.

Of the numerous illustrations which might be given, two will be selected as exemplary, namely, legal forms, and esthetic forms. The formal nature of juristic conceptions is notorious, so much so that many times during the history of law there has been good ground for complaint that forms of procedure had become the controlling factor at the expense of substance. In such cases, they ceased to be forms-of-matter and were so isolated that they became purely formalistic—a fact which perhaps contains an instructive lesson for logic, since it is clear that legal forms should be such as to serve the substantial end of providing means for settling controversies. Moreover, the objective aim is to provide in advance, as far as possible, means for regulation of conduct so that controversies are not so likely to arise. *Rules for ordering* human relations, by prescribing the ways in which transactions should be conducted, exist in order to avert conflicts, to settle them when they occur and to obtain redress for the injured party. These rules of law provide multi-

various examples of the ways in which "natural" modes of action take on new forms because of subjection to conditions formulated in the rules. As new modes of social interaction and transactions give rise to new conditions, and as new social conditions install new kinds of transactions, new forms arise to meet the social need. When, for example, a new type of industrial and commercial enterprise required large capital, the form known as limited liability supervened upon the forms constituting the legal rules of partnership.

A simpler example is found in the legal form known as contract. Agreements between persons who combine their activities for a joint end in which one person promises to do something to contribute towards reaching the end and the other person agrees to do something else, are examples of "natural" or crude modes of action. Such reciprocal engagements must have arisen at an early period in social life. But as agreements multiplied and the problem of their execution became pressing, as business became less and less a matter of direct barter and more and more a matter of agreement to exchange goods and services at a future time, certain forms arose to differentiate among kinds of reciprocal engagements. Some of them were treated as mere promises, failure to execute which brought no enforceable penalties, while others were such that failure to execute them imposed a liability upon one party and conferred an enforceable claim upon the other party.

There is nothing in the mere act of promising which differentiates one kind from the other. Certain purely formal traits had to be added to the making of a promise in order to render it enforceable; say, a seal, and evidence of a "consideration." The sum of these forms define a contract. But while the conception of contract is purely formal, it is (1) a form-of-material, and (2) it accrued to prior non-formalized material in order that the ends served by that material might be attained on a wide scale in a stabilized way. As commercial transactions became more complex, sub-kinds of contract arose, each kind of transaction having its own distinctive *formal* traits.

Men did not wait for the rise of logical theory to engage in inquiry in order to reach conclusions, any more than they waited for the law of contracts to make reciprocal promises. But experi-

ence in inquiry, as in conduct of business transactions, made it evident that the purpose for which inquiry is carried on cannot be fulfilled on a wide scale or in an ordered way except as its materials are subject to conditions which impose formal properties on the materials. When these conditions are abstracted, they form the subject-matter of logic. But they do not thereby cease to be, in their own reference and function, forms-of-subject-matter.

That the objects of the fine arts, of painting, music, architecture, poetry, the drama, etc., are what they are as esthetic objects in virtue of forms assumed by antecedent crude materials is too obvious for argument. No one acquainted with the material is at any loss to distinguish between Doric and Gothic forms in architecture or between symphonic and jazz forms in the arrangement of tonal material. Similarly, in respect to land, there are forms of record, etc., that have to be conformed to in order to give ownership a legal status. No one has any doubt about the difference between this sort of form with respect to land and that which makes a landscape an esthetic object. Poetry is marked off from prosaic description by some special form. That its material existed independently of and prior to artistic treatment, and that the relations by which that material takes on esthetic form (rhythm and symmetry for example) also exist independently, is undeniable. But it requires the deliberate effort which constitutes art, and the deliberate efforts constituting various arts, to bring the antecedent natural materials and relations together in the way that forms a work of art. The forms that result are capable of abstraction. As such they are the subject-matter of esthetic theory. But no one could construct a work of art out of the forms in isolation. Esthetic forms very definitely accrue to material in so far as materials are re-shaped to serve a definite purpose.<sup>2</sup>

2. *The Failure of Formalism.* The issue of strict logical formalism, of any theory which postulates forms apart from matter of logical forms versus forms-of-matter, comes to a head in the question of the relation to method in the natural sciences. For if formalistic logic is unable to deal with the characteristics of scientific method, a strong, if indirect, confirmation of the position

<sup>2</sup>What I have said in *Art and Experience*, in chapter VII, on *The Natural History of Form* can be carried over, *mutatis mutandis*, to logical forms.

taken in this volume is obtained. It would at first seem as if pure formalism should lead those who accept that doctrine to abstain entirely from any reference whatever to method in the natural sciences, since that method is truistically concerned with factual materials. Such, however, is not the case. Formalistic logic is not content to leave the topic of method in the existential sciences severely alone. Belief in some sort of connection is usually expressed by the phrase "logic *and* scientific method." Another expression conveying the idea of connection is the phrase "applied logic."

Both expressions serve to beg the issue, or at least to disguise the fact that there is an issue. In the case of the seemingly innocent phrase "applied logic," the real issue is whether or not the expression has any meaning at all when logic is defined in terms of forms entirely independent of matter. For the issue is precisely whether such forms *can* be applied to matter. If they cannot, applied logic is a meaningless term. For the question is not whether logical forms are applied, in the sense of being used, in inquiry into existential subject-matter, but whether they could be so used *if* they were purely formalistic. The fact that investigation into natural phenomena, when it is scientifically conducted, involves mathematical propositions, certified purely formally, may be cited, for example, as an instance of "applied" logic. The fact is not only admitted, but, as has been shown in the course of previous discussions, is necessary. The admission proves nothing, however, as to absence of relation between form and matter. It but raises the problem of the *conditions* under which the application or use of non-existential propositions in determination of propositions having material content and import takes place.

It is precisely on this fundamental matter of conditions of application that the formalistic theory breaks down. It would seem to be evident in the very nature of the case that a form which is completely indifferent to matter is not applicable to any one subject-matter rather than to another, much less capable of indicating in any selective way to *what* matter it shall be applied. If the matter in question were completely determined as formed matter *when it is given*, the problem would not arise, and it may be argued with a certain show of plausibility that such is the case in mathe-

matics. But with respect to the subject-matter of the natural sciences, no such plea can be made. Either logical forms have nothing at all to do with it (so that the question of applicability does not arise) or their application is such as to introduce into, or cause to supervene upon, the original subject-matter those properties which give it scientific standing. It is not easy to see how this supervention can take place unless logical forms are capable of somehow selecting just that specific subject-matter to which they should apply in any given scientific investigation, and are also capable of arranging or ordering that subject-matter so that conclusions of scientific validity are arrived at. For the minimum meaning that can be assigned to "application" in physical inquiry is selection (involving elimination) and arrangement. The brunt of the issue, moreover, is not faced until it is recognized that in any case the problem of *what* existential materials are to be selected and of how just *those* materials are to be ordered, is a *differential* one. For purely in the abstract, forms if they apply to *any* one subject-matter, apply equally and indifferently to *all* subject-matters, while in natural inquiries there is always the problem of determining some *special* materials in some *special* order. Whatever may be thought of this general argument, it at least serves to define what is meant by the necessity of determining the *conditions* under which pure and empty forms are applicable.

Discussion recurs, accordingly, to this problem. It is admitted that non-existential propositions, in the way of hypothetical universals, are necessary in order to arrive at fully grounded conclusions in natural science. This consideration is conclusive against traditional empiristic logic (of the type of Mill) which holds that a sufficient number of singular propositions will "prove" a generalization. But refutation of this position is far from substantiating the doctrine of the *merely* formal character of such propositions as they are used in natural science. For the crux of the problem is how in any given case the universal propositions employed acquire that *content* which is a condition of their determinate applicability. It is not enough that the propositional function "If *Y*, then *X*" should be seen to be a required form for reaching any scientifically grounded conclusions. It is necessary that *Y* should be given a *determinate value* such that *X* may also be given

a determinate value. In addition, it is an acknowledged principle that no universal proposition "implies" singulars, so that in any case there is no direct transition from universal to existential propositions. Suppose, for example, that, in some unexplained way, the purely formal "If  $Y$ , then  $X$ " has acquired content, as in the following: "If anything is human, then it is mortal." It is one thing to hold that such a proposition has directive force in instituting operations of controlled observation that determine whether any existing object has the characteristic traits describing the kind "human" from which it may be warrantably inferred that anything of this kind is "mortal." But logically it is a very different thing to hold that, apart from its operational function in instituting controlled observation, it is applicable to existence. In short, we are brought to the conclusion that *application* is a matter of existential operations executed upon existential materials, so that in the natural sciences at least, a universal proposition has a purely functional status and form.

In the above illustration, it was assumed that somehow or other the purely formal propositional function "If  $Y$ , then  $X$ ," has acquired some content so that  $Y$  has the meaning "human" necessarily related to the value "mortal." It is evident without argument that unless definite values are "insertible," the formal propositional functions have no application *even operationally* to any one existential subject-matter rather than to another. In what way then are these special values given to  $X$  and  $Y$ ? Why in a specific inquiry can we not substitute the values that would give the proposition "If angelic, then mortal"? Or, "If diseased, then immortal"? Such illustrations, capable of indefinite multiplication, make it clear that the necessary relation in question is one of *contents* having a certain *form*, not one of mere forms apart from content. The question recurs with added force: How can pure forms acquire related contents? What are the logical conditions under which they acquire those contents without which the application to existence, that marks inquiry in the natural sciences, is impossible?

Suppose that somehow the propositional form " $y\phi x$ " (or  $yRx$ ) has in some unspecified way gained enough content so that it is expressed as " $x$  is assassinated." Ignoring the problem of how the

material content "assassinated" was introduced, the question still remains why one value rather than any other of an indefinite number of possible values is given to  $x$ . It is doubtless a matter of common knowledge that Julius Caesar and Presidents Lincoln and Garfield were assassinated, and that Cromwell and George Washington were not. But how did it become a matter of public information? It would be absurd to say that it became such because of the form of the propositional function. The alternative is the obvious one that it was established by observation and record. The conception of "assassination," exclusively distinct from other modes of dying, is necessarily involved. Logically, the disjunctive form just noted, and the hypothetical proposition "If such and such differential characteristics, then this specific kind, *assassination*," are logically necessary. But they are conditions *to be* satisfied, not inherent properties; and they can only be satisfied by means of extensive and complex existential operations performed upon existential materials.

The assumption that *pure* forms constitute the required application is one more instance of confusion of the functional and directive force of a formal logical relation in prescribing conditions to be satisfied with an intrinsic structural property. Take the example frequently used in contemporary logical texts, "X is mortal," which, it is said, becomes a proposition when *Socrates* is "substituted" for X. Now either "Socrates" is here an empty symbol, devoid of content and reference, or (1) it has meaning and (2) that meaning is such as to be existentially applicable. If it is a formal symbol, nothing is gained by substituting it for X. If it has meaning in application, the meaning does not follow from the propositional function save by means of observations and observable records which determine (1) that an object, *Socrates* exists (or has existed at some definite place-time) and (2) that this object possessed the characteristics describing the kind *men*.

The propositional function "X is man" is then an expression of highly equivocal form. As soon as it is stated in its proper form (as a hypothetical universal), it is evident that operations indicated by the formula as a rule for something to be done, are necessary to determine the existence of an object satisfying the conditions laid down in the function. "X is human," in other words, formu-



lates a *problem*:—that of discovering the object or objects that are such as to possess the properties prescribed by the term “human”—a condition which requires that the meaning of “human” be already determinate. It follows that existential “application” necessarily (1) involves an existential *problem* with reference to which the *contents* of the non-existential propositions have been selected and ordered, and (2) the operational use of the formally non-existential proposition as a means of observational search for objects that satisfy the conditions it prescribes.

It is pertinent to repeat in this context the point which has been repeatedly made about doctrinal confusion of the two forms of the general proposition, namely the generic and the universal. For this confusion is absolutely indispensable for direct passage from universal propositions to propositions about a singular as of a kind and to propositions about a relation of kinds. The usual line of reasoning in support of the confusion runs about as follows: A general proposition (in the sense of generic) such as “All men are mortal,” in the sense that “Each and every man who has ever lived, who is now living or who will ever live, has died or will die” (evidently a proposition of existential import) is said, quite correctly, not to refer to any specific singular but to *any* one of an indefinite number of singulars, the existential range of which includes many singulars not now capable of observation. It affirms, in other words, a connection between the set of traits which describes *mankind* and the set of traits which describes the kind *mortal*; or those subject to the occurrence of the event, *dying*. It is also affirmed (correctly) that ultimately the warrant for asserting this connection is a proposition which affirms that the characters “*being human*” and “*being mortal*” are necessarily interrelated. Short of such a proposition, the proposition in its existential force is at best a generalization, in the sense of an *extension*, of what has been observed in some cases to an indefinite number of unobserved cases. Such an extension is “empirically” confirmed by observation of a large number of events actually occurring. But it is, in theory, open to nullification as a generalization at any time, as much so as is the proposition “All swans are white.” What takes the proposition out of this precarious form is, as a matter of fact, biological and physiological investigations which indicate a neces-

sary interrelation between the characters that *define* "living" and those which *define* "dying"—as conceptual structures.

So far there is no confusion. But the fact that the proposition "All men are mortal" does not refer to any *specified* singular as such, or to any one man rather than to any other, is illegitimately interpreted to mean that it does not refer to *any* singular whatever. The proposition is then converted into the non-existential proposition, "If human, then mortal." The conversion is illegitimate because it is one thing, logically, to make propositions about traits or characteristics which describe a kind in "abstraction" from any *given* singular of the kind, and a radically different thing to make a proposition about abstractions *qua* abstract. The absence of specific reference to one singular rather than another is no ground for a proposition free from *any* existential reference. There is no logical road from "No *specific* singular" to "No singular *whatever*," in the sense of abstraction from existential reference as such. Yet this is the impossible road taken by logical doctrine when it assimilates the form of generic propositions to those of universals.

The fact that in the context of discussion of logical forms it is expressly pointed out that singular and generic propositions—all propositions of the *I* and *O* form—have existential reference, while no universal of the *A* or *E* form has existential reference, shows that the confusion in question is not an accidental slip, or a case of occasional carelessness. The confusion is inherently essential to any doctrine which (1) holds that logical forms are formal in the sense of being independent of content, factual or conceptual, and yet (2) are capable of material application—as is inherently involved in the methods of the natural sciences if the latter have any connection at all with logic. In spite of the appearance of the word *all* in the proposition "All men are mortal" (as a proposition referring to each and every singular of the kind described by the sets of distinguishing traits that determine respectively the kinds "human beings" and "subject to death"), the proposition is logically an *I* proposition—a fact recognized in the doctrine expressly stated (in another context) that *I* and *O* propositions alone refer to existence.<sup>3</sup>

<sup>3</sup> A conspicuous case of the confusion in question is found in the treatment of the null class. That the kinds *Indian Popes*, *Emperors of the United States*,

I recur to the earlier statement that while scientific method is not possible without non-existential *if-then* propositions, and while such propositions are necessary conditions of scientific method, they are not its *sufficient* conditions. An hypothesis concerns what is *possible*, and a proposition regarding possibles is indispensable in inquiry that has scientific standing. The hypothesis is formulated in an abstract *if-then* proposition. It then formulates a rule and method of experimental observation. Consequences of the execution of the indicated operations define *application* in the only logically coherent sense of that conception. One indispensable *condition* of application in the case of method in natural science is, therefore, that the *contents* of the hypothetical proposition be themselves determined by prior existential inquiries in such a way that the contents are capable of directing further operations of observation. Moreover, even in such cases, the fallacy of affirming the antecedent because the consequent is affirmed is committed, unless independent operations of extensive observation have affected the affirmed relation of contents with a *probability coefficient*. The validity of any such coefficient is conditioned upon the nature of other existential propositions and their material consequences.

The case of ordered discourse in which all propositions are, as such, non-existential in import, and which form a series in virtue of the implicatory, as distinct from the inferential, function affords at best but a seeming exception to the principle that forms are forms-of-matter. For the sequential order of any such series is determined in all cases, in which the final proposition has applicability, by material conditions. Theoretically or in the ab-

have no members, and are instances of a "null class" is a statement having radically different logical form from expressions like, say, *circular-square*, or *vicious-virtue*. The first is an instance of contingency; up to a given date, no such singular has existed or, if it has existed, has not been observed. The second set of examples express necessary exclusion of an instance, since the related conceptions contradict each other. The case of vicious-virtue is perhaps especially instructive. There can be no doubt that there occur actions which conventionally are called virtuous but which, from the standpoint of some ethical theories, are inherently vicious and vice versa. This fact does not signify that the definitions of vice and virtue are compatible with each other, but that from the standpoint of one ethical theory the definitions of vice and virtue held by its adherents are incompatible with the conceptions held by adherents of another ethical theory.

stract, an indefinite variety of series of implicatory propositions is possible—as in mathematics. But—as appears in mathematical *physics*—mathematical implicatory series, in all instances in which applicability enters as a condition, have their contents and their order (in determining a final hypothetical proposition) controlled by the observed existential conditions that form the problem requiring a generalized solution. Otherwise, contents would be taken and ordered in such an indeterminate way that even if the order were necessary with respect to rigor of implication, there would be no assurance whatever of any kind of final applicability. We are again forced to the conclusion that formal relations state conditions to be materially satisfied.

The arguments adduced show incontestably that pure forms, where “pure” means “completely independent of relation to meaning-contents” (factual and conceptual) cannot possibly determine application in the sense in which application is necessary in the natural sciences. There is one especial instance frequently given in recent logical treatises which is supposed to prove that a universal proposition is capable of direct determination of an inference regarding existential matters. It is, accordingly, worth examining, since this will disclose the typical fallacy involved in all instances of the doctrine in question. The example referred to is the following: From the *if-then* proposition “If there are more inhabitants in a town than there are hairs on the head of any inhabitant of that town, then some two (or more) inhabitants have the same number of hairs on their heads.” There is, of course, no possible doubt that *if* the conditions stated in the antecedent clause are satisfied, then the state of affairs set forth in the consequent clause will follow. But as far as an *existential* proposition about a person or persons in any actual town are concerned, the proposition only raises a question: Are the conditions satisfied?

This question is one of material fact. It can be answered only by independent operations of observation that are directed by the *if-then* proposition in question. This proposition, when so employed, renders it unnecessary to count the hairs on the head of every person in a town. It is necessary only to have a dependable estimate of the number of hairs on the head of the bushiest-haired person that can be found and also have a dependable estimate of

the number of inhabitants of the town. Given these existential data, the inferred proposition that *some* two persons do (or do not) have the same number of hairs on their heads will be warranted. The conclusion that they do *not* have, would be more likely in the case of a hamlet where there are only a few inhabitants. Observational data would suffice in the case of a very large city like London or New York to warrant the existential proposition that two or more (unspecified) persons do have the same number of hairs on their heads. But it would do so not because that proposition is "implied" by the hypothetical proposition in question, but because of determination by observations of existential data, taken in connection with the hypothetical proposition as the rule for their selection and ordering.

A similar mixing of propositions of two different logical forms is found in the notorious case of Epimenides and Cretans as liars. Epimenides who is a Cretan, according to an existential proposition, affirms that "All Cretans are liars." Hence, it is argued, a contradiction or "paradox" inevitably arises. Unless Epimenides speaks the truth, it does not follow that All Cretans are liars, and if he speaks the truth, then the proposition follows that "Some Cretans tell the truth" and hence the proposition that "All Cretans are liars" is false. Only a little analysis is required to show that if the proposition "All Cretans are liars" is a *generic* proposition, meaning that a disposition to lie is one of the characteristic traits that mark off Cretans as a kind from other kinds of Greeks (or of human beings), it does not follow that every Cretan is *necessarily* a liar and that he always lies. For the trait of lying describes a Cretan only in conjunction with other circumstantial, or temporo-spatial, conditions which are contingent since they are existential. In other words, if the proposition is generic, some Cretan may sometimes tell the truth, and there is no contradiction. On the other hand, if the ambiguous term "all" is interpreted in the sense of a *necessary* relation between *being a Cretan* and *being a liar*, or as the contents of a universal, instead of a generic, proposition, it poses a question as far as any existential proposition is concerned. If Epimenides tells the truth when he says "All Cretans are liars" then by definition he is not in fact a Cretan. For denying the consequent denies the antecedent. If on the other hand, it is

found, by adequate observation of existential data, that he is lying then it is necessary to revise the hypothetical universal proposition in question—a state of affairs that always occurs when application of a universal proposition to existential condition is found to yield data that are not in accord with the requirements of the universal. The conclusion of the analysis is that only a mixing of the two forms, the generic and the universal, produces the alleged contradiction.<sup>4</sup>

Exactly the same analysis applies to the existential conclusion that in a country having a monogamous system, it can be inferred that the number of husbands and wives is equal without having to go through the tedious process of enumeration of the actual number of husbands and wives. For independent operations of observation are required to determine whether a given country has or has not a monogamous system. The same holds in the case of the inference that in a given hall the number of seats and the number of persons may be determined to be equal without counting the number of either. For again, it requires independent observation to determine that every seat is in fact occupied. The source of fallacy in all these instances is that, first, cases are taken whose materials have been *prepared* by prior existential operation, and, secondly, that the way in which they were prepared is ignored, the ignoring being here equivalent to denial.

The discussion, so far, has supported the doctrine that logical forms are forms-of-matter on a negative ground: namely, the contradictions that exist upon the alternative basis. The positive support of the doctrine is the fact that in scientific inquiry, specific contents, factual or conceptual, as well as forms in which they are ordered, are determined in strict correspondence with each other. An attempt to justify this statement at this juncture would be to repeat the analyses and conclusions of the whole two previous Parts. Instead of engaging in this superfluous task, the point of issue will be approached by consideration of the principle that is present in analogous subject-matters. The basic category of

<sup>4</sup> A similar analysis applies to the alleged paradox of the "autological" and the "heterological." In one set of propositions, the words have to do with a conception or a category and in the other case with a *word*, which is existential. These "paradoxes" occur only when the logical ambiguity of "class" (as meaning both kind and category) is taken advantage of.

logic is *order*. It is also the basic category of all the arts. The universal order of material contents in every intelligently directed procedure is that of means-to-consequences; actual existential materials providing the "stuff," while the status of the material *as* means requires operations of selection and of re-arrangement so that special interactions may be instituted to effect the consequences intended. At the outset, when a certain result is desired, some existing material in its "natural" or crude state may be used—as a stick conveniently at hand is used to pry a stone. In such a case, required operations of observation are directed merely to selection of a suitable stick. But when need for a certain kind of consequence is recurrent, it becomes advisable to select just the materials that lend themselves to formation of the tools that most expeditiously and economically effect the intended end in a great variety of temporal-spatial circumstances. Materials are then selected and shaped to be *levers*. At a certain level of culture, the lever may be simply a crowbar. But as need develops that consequences be brought about in a widened variety of circumstances, the principle of leverage is expanded and refined to include a variety of physical devices, which, scientifically stated, avail themselves of the law of momenta to obtain mechanical "advantage." An expert mechanic thus becomes acquainted, even apart from comprehension of a scientifically formulated law, with a variety of contrivances all of which are levers because, in spite of their different sizes and shapes, they have the functional relation of being means to a specified distinctive kind of consequence.

Every tool, appliance, article of furniture and furnishings, of clothing, every device for transportation and communication, thus exemplifies practically and existentially the transformation of crude materials into intentionally selected and ordered means so that they are *formed-matter*; or, stated from the side of form, so that there are forms-of-matter. Form and matter may become so integrally related to one another that a chair seems to be a chair and a hammer a hammer, in the same sense in which a stone is a stone and a tree is a tree. The instance is then similar to that of the cases in which prior inquiries have so standardized meanings that the form is taken to be inherent in matter apart from the function of the latter; or (as in the case of some of the formalistic argu-

ments which have been criticized) matter is treated as if it were itself purely formal—a conclusion drawn because integration of form and matter is so completely accomplished.

These instances exemplify the principle stated in the first part of this chapter; namely, that forms regularly accrue to matter in virtue of the adaptation of materials and operations to one another in the service of specified ends. They are here brought forward, however, for a different but related purpose—namely, to illustrate the principle that in all cases of formed-materials, form and matter are instituted, develop and function in strict correspondence with each other. Every tool (using the word broadly to include every appliance and device instituted and used to effect consequences) is strictly *relational*, the relational form being that of means-to-consequences, while anything which serves as effective means has physical existence of some sort.

1. The abstract relation of means-consequence may be formally analyzed. It involves correspondence of material and procedural means, a correspondence illustrated, in the field of tools, utensils, articles of dress, etc., in the fact that materials and techniques are reciprocally adapted to each other. Technical processes of reshaping raw materials are invented so that they are capable of reshaping the crude material to which they are applied to make the latter function as means. The processes must be such as to be capable of just those modes of application that are suited to the materials with which they deal. Techniques once initiated are capable of independent development. As they are perfected, they not only transform old materials expeditiously and economically, but they are applied to crude materials previously not capable of use as means. The new formed-matter thus produced leads to further development of techniques and so on indefinitely, with no possibility, from the theoretical side, of setting a limit.

2. Any technique or set of procedural means must satisfy certain conditions of order so that it possesses *formal* properties. The crudest technical procedure in reshaping crude material has, of necessity, a definite *initiation*, *termination* and an *intermediation* that connect the two limits. It has the formal properties of first, last and intermediates—the latter being so essential as to define even the word “*means*.” The ordered transitive relation of *first*,



*last* and *middle* operations is *formal* and capable of abstraction because it constitutes a necessary *interrelation* of characters. Change any one of them and the others are necessarily also modified. Generalize the point here made and there emerges the conception of *serial order* as an order necessary to matter *qua* formed-matters from the point of view of all intelligent activity.

3. Because of the first point mentioned (the conjugate correspondence of material and procedural means or techniques), the serial order of procedure determines formal relations in the materials to which the techniques are applied. Even the crude primitive techniques employed to effect objective consequences brought about a crude differentiation between characteristic properties of materials. Certain materials were found to be "good for" the techniques by which clothing was produced; other materials for making utensils in which to store or to cook materials, etc. As techniques of smelting developed, characteristic differences in mineral materials were automatically, so to say, noted in such a way as to mark off different kinds of metals. The principle here exemplified is generalized in the statement that differential characteristics, describing different kinds, are instituted when and only when materials are adjudged as means in connection with operations to accomplish specified objective consequences. An accomplished end, say, clothing, is generic. But it comes about that different kinds of clothing are appropriate to different seasons, occasions, and social castes. Different materials are such as to be "good for" these differential ends: one kind for winter, another for summer; one for war and another for peace; one for priests, another for chiefs, and another for "common" people. Kinds are distinguished and related in strict correspondence with each other.

Were we to recur to the considerations adduced in the chapter on the biological matrix of inquiry, we should note that the formal relations of serial order are prefigured in organic life. There are needs (in the sense of existential tensions); these needs can be satisfied only through institution of a changed objective state of affairs. Effectuation of this close, or consummatory state, demands an ordered series of operations so adapted to *one another* that they are co-adapted to arriving at the final close. If we com-

pare these natural organic cases of means in ordered relation to a consequence one difference of importance appears. The "end" in the case of the relation of activities and material conditions is, in the case of the former, an end in the sense of a close or termination. In the case of the latter, there is an *additive* character. The objective close in being foreseen and intended, becomes an end-*in-view* and thereby serves to *direct* intelligent selection and arrangement of techniques and materials. But there is a common pattern of relationship.

Upon the practical side, the considerations which have been brought forward are so familiar as to be commonplaces. They may seem, therefore, not worth noticing in the discussion of logical theory. But they are pertinent because they bring out a number of points that are fundamentally significant in logical theory. The main considerations may be recapitulated as follows: (1) The accruing of forms to matter in the case of inquiry is not a gratuitous hypothesis. (2) Whenever materials become formed-materials, there is involved a definite order, the serial. (3) This order, being formal, may be abstracted and be formulated in such a way that its implications are developed in discourse. (4) There is continuity of development from the orderly relations of organic life through the deliberately ordered relations of the cultural arts to these characteristic of controlled inquiry.

It is important, in this connection, not to confuse the categories of potentiality and actuality. Crude materials must possess qualities such as permit and promote the performance of the specific operations which result in formed-matter as means to end. But (1) these qualities are but potentialities, and (2) they are *discovered* to be the potentialities which they are only by means of operations executed upon them with a view to their transformation into means-to-consequences. At the outset, these operations of transformation may be random and "accidental." In the progress of culture, they become so controlled that they are experimental in the scientific sense of that word. The first point is illustrated in the fact that with the emergence of animal life certain materials became *foods*. We may then say that these materials *were* foods all the time and even that they are intrinsically or "by nature" *foods*. Such a view confuses potentiality with actuality. Looking back,

we can validly affirm that these materials were *edible*. But they are not foods in actuality until they are *eaten and digested*, i.e., until certain operations are performed that give crude materials those new properties which constitute them of the special kind *foods*. The second point is illustrated in the fact that the difference between edible, non-edible, and poisonous properties was discovered only by processes of trying and testing. Even tribes regarded as primitive have found ways of instituting technical operations which transform stuffs that are poisonous in their crude state into means of nourishment. That qualities *qua* potentialities are ascertained by experimental operations is proved by the fact that with extension and refinement of physico-chemical operations the range of things that are edible has been indefinitely extended. Whether, for example, the attempt to produce milk "artificially" will succeed or not is wholly a matter of available techniques, not a theoretical matter save in the sense that some theory is necessary to *guide* practical effort.

This relativity to operations of the qualities that constitute the characteristic traits which describe kinds, together with the relativity of the discovery of the latter to execution of operations, is fatal, as we saw earlier, to the classic doctrine that inherent natures or essences define kinds. But it has another important bearing upon logical theory. The previous discussion has been limited to doctrines that make a sharp separation between form and matter. But there still exist logical theories which assign *direct* ontological status to logical forms, although in a different way from that of Aristotelian logic. These theories rest upon a basis of fact. For they recognize that logical forms can apply to existential material only in a thoroughly ungrounded, external, arbitrary fashion unless material as existential has its own intrinsic capacity for taking on these forms. But this valid insight is misinterpreted by means of the precise confusion of potentiality with actuality just mentioned. Existence in general must be such as to be *capable* of taking on logical form, and existences in particular must be capable of taking on *differential* logical forms. But the operations which constitute controlled inquiry are necessary in order to give actuality to these capacities or potentialities.

The particular way in which in recent theory, logical forms

are given *direct* existential status (instead of indirect status through their functions in inquiry) is a metaphysical interpretation of *invariants*. The use of certain mathematically formulated constants in physical inquiry is an illustration, as far as it goes, of the meaning of an invariant in its logical sense. If we generalize what is involved, logical forms are invariants. For example, there is no ordered discourse apart from the implicatory relation as a constant, and there is no grounded inference apart from the invariant formal relation of conjunction of traits to descriptive determination of kinds. But it does not follow from the fact that "invariants" are necessary for the conduct of inquiries that yield warranted knowledge that they are therefore necessary in and to the *existence* which knowledge is *about*. Under the guise of a valid principle, namely, that logical forms have existential reference, there is slipped in quite another principle; namely, a particular metaphysical conception about existence, and this special *preconception* is then used to settle the meaning of logical invariants. Thereby logic is rendered heteronomous; or dependent upon a metaphysical principle not itself arrived at by logically determined methods. In scientific procedure, moreover, an invariant is such in relation to a specified set of operations, while the view criticized assumes that invariants are such absolutely.

The external character of the metaphysical assumption is strikingly evident in that by definition it concerns existence, whereas inquiry into existence can only arrive at conclusions having a coefficient of some order of *probability*. And it is obvious that conceptions of a *probable* invariant and an immutable structure are self-contradictory. Moreover, the conception is gratuitous. For the necessary operational presence of invariant forms in arriving (through inquiry) at warranted conclusions is completely explicable on the ground of competently controlled conduct of inquiry itself. Assumption of a one-to-one correspondence between the forms of authentic knowledge and the forms of existence does not arise from necessary conditions within the logic of inquiry. It proceeds from some outside epistemological and metaphysical source.

The net conclusion of both the critical and constructive portions of the foregoing discussion is that the phrase "logic *and*

scientific method" has no valid meaning when "and" is taken to mean an external relation between the two terms. For scientific method both constitutes and discloses the nature of logical forms. It constitutes them in the actual practice of inquiry; once brought into being, they are capable of abstraction:—of observation, analysis and formulation in and of themselves. In connection with this conclusion, it is pertinent to summarize briefly the outcome of some previous discussions.

1. The history of actual scientific advance is marked by the adoption and invention of material devices and related techniques:—of complex and refined forms of apparatus and definite related techniques of using apparatus. Even in the last half-century, astronomical science has been revolutionized by invention and use of such material instruments of inquiry as the spectroscope, bolometer, ultra-violet glass, chemical emulsions in photography, use of aluminum instead of mercury for coating mirrors, and the techniques which have made possible construction of lenses eighty inches in diameter and mirrors having a diameter of two hundred inches.<sup>5</sup>

2. The new data thus instituted do much more than provide facts for confirming and refining old conceptions. They institute a new order of problems whose solution requires a new frame of conceptual reference. In particular, it was by the use of new instruments and techniques that changes and relations of change were disclosed in what had previously been taken as fixed; a process that has gone on at an accelerated rate since the seventeenth century. This change in the nature of data was both the source and the product of the universal adoption of the experimental method and of the new order of conceptions demanded by its successful execution.

3. Upon the conceptual side, this scientific revolution was accompanied by a revolution in mathematical conceptions; again,

<sup>5</sup> Cf. *ante*, pp. 252-3. The following passage is worth citation as one of the comparatively few instances of recognition, from the side of theory, of the importance of this point: "The reason why we are on a higher imaginative level [in science] is not because we have finer imagination but because we have better instruments. In science, the most important thing that has happened in the last forty years, is the advance in instrumental design. . . . These instruments have put thought on a new level." A. N. Whitehead, *Science and the Modern World*, p. 166.

partly as cause and partly as effect. As long as Euclidean geometry was taken to be the exemplary model of mathematical method, the underlying categories of mathematics were such as to be applicable only to structures fixed within certain limits. The logic of deduction from first, and immutable truths then remained supreme, wherever the necessity for general principles was acknowledged. Cartesian analytics, the calculus, and subsequent developments were called for by the radically new emphasis placed in scientific inquiry upon correlations of change, while independent development of mathematical conceptions disclosed in their application to existence new, more refined and extensive problems of correlated change.

In the meantime, the development of a genuinely empirical theory of logic, one in accord with actual scientific practice, was seriously retarded and deflected by adherence to an order of ideas that had developed in the prescientific epoch. The incompatibility of this conceptual framework with the actual procedures and conclusions of scientific inquiry strengthened, by reaction, the position of the non-empirical *a priori* school. Mill's logic, as representative of the earlier type of empiricism, is a noteworthy combination of genuine concern for scientific method as the sole source of valid logical theory with a misinterpretation of that method, which is due to an adherence to notions about sensations, particulars, and generalizations, that were formulated before the rise of modern scientific method. The outcome was his denial of the importance of conceptions; reduction of hypotheses to a secondary "auxiliary" position; the idea that mere particulars can "prove" a generalization, etc.

This chapter, in both its critical and constructive phases, is, then, preparatory to detailed examination of the logic of scientific method, as that is exhibited in the mathematical and in the natural sciences. In a certain sense, the order of exposition of topics in this treatise is the reverse of the order in which their contents have actually developed. For, as just indicated, the special logical interpretations which have been advanced represent the conclusions of analysis of the logical conditions and implications of scientific method, while in the interpretations that form the previous chapters they have been taken up, for the most part, on the

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ground of their logical status as such. The following chapters thus serve both as an explicit formulation of the ultimate foundations of the views previously expressed, and as a test of their validity.

Because of the critical role of mathematics in physical science and because also of the peculiarly formal character of mathematical subject-matter, the logical conditions of mathematical discourse will be taken up first in order of the topics that are discussed.